SECTION 6.4: VOLUME BY CYLINDRICAL SHELLS

RECALL: To find the volume of a solid of revolution using the disk/washer method, we chop up the axis of rotation. In this section, we investigate chopping up the other (perpendicular) axis. If we imagine rotating a representative rectangle around a perpendicular axis, the resulting shape is called a **cylindrical shell** with volume:

$$V_{\text{shell}} = 2\pi\,(\text{average radius})\,(\text{height})\,(\text{thickness})$$

In our scenario, the 'average radius' of the shell is the distance from the shell to the axis of rotation. The 'height' of the shell is the height of the representative rectangle. Finally, the 'thickness' of the shell is the differential of the variable representing the axis we are chopping up, dx or dy.

EXAMPLE 1: Let R be the region in Quadrant I bounded by $y = 4 - x^2$, $y = 0$, and $x = 0$.

Use the 'Shell Method' to find volume of the solid obtained by rotating R about

1. the y-axis.

$$\text{Ans: } V = \int_0^2 2\pi\, x\left(4 - x^2\right)\, dx = \ldots = 8\pi \text{ units}^3$$

2. the line $x = 3$.

$$\text{Ans: } V = \int_0^2 2\pi\,(3 - x)\left(4 - x^2\right)\, dx = \ldots = 24\pi \text{ units}^3$$

3. the line $x = -1$.

$$\text{Ans: } V = \int_0^2 2\pi\,(x + 1)\left(4 - x^2\right)\, dx = \ldots = \frac{56\pi}{3} \text{ units}^3$$

EXAMPLE 2: In the last section, we used the washer method to find the volume of the solid obtained by revolving the region bounded by $y = x^2$ and $y = \sqrt{8x}$ about the y-axis. Find this volume using the shell method.

$$\text{Ans: } V = \int_0^2 2\pi\, x\left(\sqrt{8x} - x^2\right)\, dx = \ldots = \frac{24\pi}{5} \text{ units}^3$$

EXAMPLE 3: Find the volume of the torus obtained by rotating the region bounded by $(x - r)^2 + y^2 = R^2$ about the y-axis. Assume $r > R > 0$.

$$\text{Ans: } V = \int_{r-R}^{r+R} 2\pi\, x\left(2\sqrt{R^2 - (x - r)^2}\right)\, dx = \ldots = 4\pi \int_{-R}^{R} (u + r)\sqrt{R^2 - u^2}\, du = \ldots = (2\pi r)(\pi R^2) \text{ units}^3$$

HOMEWORK: Section 6.4: 9 - 57 odd; Section 8.2: 43, 45, 47